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Two-Loop Electroweak Corrections to the Muon $g-2$: a new class of Hadronic Contributions

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Abstract

We discuss, within the framework of the Standard Model, the calculation of the two-loop electroweak contributions to the anomalous magnetic moment of the muon involving triangle fermionic loops of leptons and quarks. Because of the large ratios of masses involved, these contributions are rather large. The result we obtain differs from a previous estimate reported in the literature. The discrepancy originates in the cancellation of anomalies in $SU(3)_c \times SU(2)_L \times U(1)_Y$, a cancellation that requires the consideration of both leptons *and* quarks within each generation and that had been previously overlooked.

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1. There is a forthcoming experiment at the Brookhaven National Laboratory which plans to measure the anomalous magnetic moment of the muon with an expected uncertainty of $\pm 40 \times 10^{-11}$. This will correspond to an improvement by a factor of *twenty* with respect to the latest result obtained from the experiment performed at CERN, which gave [1]:

$$a_\mu \equiv \frac{1}{2}(g_\mu - 2) = 11659230(85) \times 10^{-10}. \quad (1)$$

The muon $g - 2$ project at BNL [2] has renewed the interest on the part of some theorists to improve the accuracy of the corresponding prediction in the Standard Model. Here we want to concentrate on the electroweak higher-order corrections. The history and the present status of the calculation of the dominant electromagnetic contributions to a_μ can be found in the series of review articles [3], [4], and [5]. A recent phenomenological re-evaluation of the hadronic vacuum polarization effect on a_μ has been made in ref. [6]. The hadronic light-by-light scattering effect on a_μ has also been recently reconsidered, within the framework of low-energy QCD, in refs. [7], [8] and [9].

The one-loop contributions to a_μ due to the electroweak interactions of the Standard Model were calculated quite a long time ago [10]. The relevant Feynman diagrams are shown in Fig.1. The corresponding result is

$$a_\mu^{\text{Weak}} = \frac{G_F m_\mu^2}{\sqrt{2} 8\pi^2} \left\{ \frac{10}{3} + \frac{4}{3}(v_\mu^2 - 5a_\mu^2) + \mathcal{O}\left(\frac{m_\mu^2}{M_Z^2} \log \frac{M_Z^2}{m_\mu^2}\right) + 2 \int_0^1 dx \frac{x^2(2-x)}{x^2 + \frac{M_H^2}{m_\mu^2}(1-x)} \right\}, \quad (2)$$

where v_μ and a_μ are the vector and axial-vector couplings of the Z to the muon. In general, for a fermion f ,

$$v_f = I_f^{(3)} - 2Q_f \sin^2 \theta_W, \quad a_f = I_f^{(3)}. \quad (3)$$

The contribution from the Higgs, given in terms of a parametric integral in eq. (2), decouples in the infinite mass limit. Numerically, with the Higgs contribution neglected,

$$a_\mu^{\text{Weak}} = 195 \times 10^{-11}. \quad (4)$$

The leading two-loop electroweak contributions to a_μ have been discussed in ref. [11]. These authors have selected all the possible sources of logarithmically enhanced terms which appear because of the large ratios of masses involved, and which result in contributions to a_μ of order

$$\mathcal{O}\left(\frac{G_F m_\mu^2}{\sqrt{2} 8\pi^2} \frac{\alpha}{\pi} \log \frac{M^2}{m^2}\right), \quad (5)$$

with $M \gg m$, where typically M is the Z mass and m a fermion mass, like for instance the muon. Apart from these, other possible logarithms involving the Higgs mass like $\log M_H/M_Z$ or $\log M_H/m_t$ might also appear but, unless $M_H \gg m_t$, they are not so large and are disregarded. The authors of ref. [11] thus find the following overall correction $\Delta a_\mu^{\text{Weak}}$ to the one-loop result in eq. (4):

$$\Delta a_\mu^{\text{Weak}} \simeq -42 \times 10^{-11}; \quad (6)$$

i.e. a rather large negative correction, of the same size as the expected experimental uncertainty [2].

2. We shall be concerned with a specific class of the two-loop electroweak contributions: those induced by virtual fermionic triangle loops, represented by the Feynman diagrams in Fig. 2. The authors of ref. [11] have only considered the subclass of these contributions where the fermion in the triangle loop is a *lepton*. In the 't Hooft-Feynman gauge and keeping only the asymptotic contributions from the large ratios of masses involved, the results they find are:

$$\Delta a_\mu^{\text{Weak}}|_e \simeq -\frac{G_F}{\sqrt{2}} \frac{m_\mu^2}{8\pi^2} \frac{\alpha}{\pi} \left[3 \log \frac{M_Z^2}{m_\mu^2} + \frac{5}{2} \right] = -11.7 \times 10^{-11}; \quad (7)$$

$$\Delta a_\mu^{\text{Weak}}|_\mu \simeq -\frac{G_F}{\sqrt{2}} \frac{m_\mu^2}{8\pi^2} \frac{\alpha}{\pi} \left[3 \log \frac{M_Z^2}{m_\mu^2} - \frac{8}{9} \pi^2 + \frac{11}{6} \right] = -9.11 \times 10^{-11}; \quad (8)$$

$$\Delta a_\mu^{\text{Weak}}|_\tau \simeq -\frac{G_F}{\sqrt{2}} \frac{m_\mu^2}{8\pi^2} \frac{\alpha}{\pi} \left[3 \log \frac{M_Z^2}{m_\tau^2} - 6 \right] = -4.77 \times 10^{-11}. \quad (9)$$

One should realize that the results of eqs. (7-9) are actually gauge dependent. These results, as they stand, stem from the $g_{\mu\nu}$ part of the Z propagator in the diagrams of Fig. 2. [‡] In the unitary gauge, for instance, the $k^\mu k^\nu$ piece in the Z propagator yields an extra contribution to eqs. (7-9) that has a part that is *common* to these three equations. This extra common contribution is actually *divergent* and originates in the anomaly that results when one multiplies the triangle by k^μ . It is only when one sums over a complete generation that the anomaly vanishes and the result is finite and gauge invariant.[§] As a consequence, strictly speaking, only the contribution of a full generation may be considered physically meaningful, but not that of a single fermion.

The Feynman diagrams in Fig.2 where the fermion in the triangle loop is a *quark* correspond to a new class of *hadronic contributions* to the muon anomaly. We shall call them the hadronic Z - γ - γ contributions. The fact that the fermionic triangle subdiagram in Fig. 2 has an Adler, Bell-Jackiw VVA anomaly, which in the Standard Model cancels when all the fermions of the same generation are included, implies the vanishing of the whole triangle diagram in the limit of exact mass degeneracy within each generation. We therefore expect important cancellations within each generation, which questions the overall estimate in eq. (6) quoted from ref. [11].

We propose to investigate this problem first within the framework of effective quantum field theories where the underlying physics can be easily understood. We have also done an exact calculation of the contribution to a_μ from the diagrams in Fig. 2, and checked that, in the appropriate cases, the various asymptotic limits reproduce the simple effective field theory expressions. We have reproduced in particular the results in eqs. (7), (8), and (9) in the 't Hooft-Feynman gauge. We shall then discuss the new numerical results.

3. Let us first consider the limit where the Z -mass is much larger than the masses of any of the other particles involved, which is certainly the case for the first and second generations,

[‡]The associated diagrams with a would-be Nambu-Goldstone have an extra suppression due to the mass of the fermion going around the triangle, m_f , that goes like m_f^2/M_Z^2 .

[§]The gauge invariance of the Standard Model is a fact only after the cancellation of anomalies is effected.

and keep only the leading $\log M_Z$ contributions. When the Z -field in the Standard Model is integrated out, there appear new local four-fermion couplings induced by the tree-level exchange of the underlying Z propagator. These four-fermion couplings lead to two-loop diagrams like the ones shown in Fig.3, which yield contributions to the muon $g - 2$ that are logarithmically divergent. This divergence is to be interpreted as cut off by M_Z . If furthermore the fermion masses in the triangle loop are neglected with respect to the muon mass, the result from each fermionic contribution is the same, up to a factor $Q_f^2 a_f$ proportional to the square of the electric charge Q_f of the fermion times its axial coupling a_f defined in eq. (3). For quarks, there is an extra factor of *three* from the number of colours. The overall result we find for the muon $g - 2$ from the first generation, in the limit where $m_e = 0$, and in the chiral limit where $m_u = m_d = 0$, is then:

$$\Delta a_\mu^{\text{Weak}}|_{e,d,u} \simeq \frac{G_F}{\sqrt{2}} \frac{m_\mu^2}{8\pi^2} \frac{\alpha}{\pi} \sum_f Q_f^2 a_f \left(6 \log \frac{M_Z^2}{m_\mu^2} + \frac{\mathcal{A}}{\epsilon} + \mathcal{B} \right) = 0, \quad (10)$$

where the $1/\epsilon$ term encodes the ultraviolet divergences in dimensional regularization and the m_f -independent constants \mathcal{A}, \mathcal{B} are gauge dependent. They vanish in the 't Hooft-Feynman gauge but not in the unitary gauge. Each fermion contribution is separately gauge dependent and it is only the sum over the full generation that is physically meaningful. The fact that $\sum_f Q_f^2 a_f = 0$ is of course a property due to the anomaly cancellation in the $SU(3)_c \times SU(2)_L \times U(1)_Y$ gauge theory. We shall later come back to a more elaborate discussion of the contribution from the first generation, which takes into account the effect of hadronic mass scales in the light quark sector of QCD.

4. The evaluation of the leading contribution from the second generation of fermions is a little more delicate. We can still consider the effective field theory where the Z -field has been integrated out, but now the fermion masses in the triangle loop cannot be neglected with respect to the external muon mass. The contribution from the strange quark, in particular, requires a special discussion depending on whether or not one is willing to consider the chiral $SU(3)$ limit where $m_s = 0$, and on whether or not one wants to discuss spontaneous chiral symmetry breaking effects and the effects due to the QCD $U(1)_A$ anomaly. To a first approximation, we shall also consider the chiral limit for the strange quark and, as in the case of the first-generation estimate, we shall neglect for the time being the effect of hadronic mass scales. We then find :

$$\begin{aligned} \Delta a_\mu^{\text{Weak}}|_{\mu,s,c} &\simeq \frac{G_F}{\sqrt{2}} \frac{m_\mu^2}{8\pi^2} \frac{\alpha}{\pi} \left\{ \left(-\frac{1}{2}\right) 6 \log \frac{M_Z^2}{m_\mu^2} + 3 \frac{1}{9} \left(-\frac{1}{2}\right) 6 \log \frac{M_Z^2}{m_\mu^2} + 3 \frac{4}{9} \left(\frac{1}{2}\right) 6 \log \frac{M_Z^2}{m_c^2} \right\} \\ &= -\frac{G_F}{\sqrt{2}} \frac{m_\mu^2}{8\pi^2} \frac{\alpha}{\pi} 4 \log \frac{m_c^2}{m_\mu^2} \simeq -5.4 \times 10^{-11}, \end{aligned} \quad (11)$$

where the numerical result is for $m_c = 1.3 \text{ GeV}$. As expected on first principles, the Z -mass does not appear in the final result.

5. The evaluation of the leading contribution from the third generation brings in an interesting issue, related to the fact that the top quark is heavier than the Z . Within the

framework of effective field theories, we now have to consider the case where the top field is integrated out first, corresponding to the limit $m_t \gg M_Z$. In this limit, the top quark in the $Z\text{--}\gamma\text{--}\gamma$ vertex decouples. The corresponding effective local $Z\text{--}\gamma\text{--}\gamma$ coupling induced by the top triangle loop goes as $1/m_t^2$ and induces a contribution to the muon $g-2$ at the one-loop level via the Feynman diagrams shown in Fig. 4. If we work in the limit $m_t \rightarrow \infty$, this contribution vanishes. In this limit and in the unitary gauge (where there are no would-be Nambu-Goldstone bosons) the integration of the top leaves no trace behind in the form of new local effective operators relevant to the $g_\mu - 2$.[¶]

In the effective Lagrangian without top the $g_{\mu\nu}$ part of the Z propagator yields for τ and b in the loop a finite contribution completely analogous to that found in the previous case for the first two generations. The $k^\mu k^\nu$ part, on the other hand, yields the anomaly when multiplied by the triangle and this contributes a logarithmically divergent quantity that is to be interpreted as cut off by the top mass. Gathering all the pieces we obtain:

$$\begin{aligned} \Delta a_\mu^{\text{Weak}}|_{\tau,b,t} &\simeq -\frac{G_F}{\sqrt{2}} \frac{m_\mu^2}{8\pi^2} \frac{\alpha}{\pi} \left\{ 3 \log \frac{M_Z^2}{m_\tau^2} + \log \frac{M_Z^2}{m_b^2} - \sum_{f=\tau,b} Q_f^2 a_f 4 \log \frac{m_t^2}{M_Z^2} \right\} \\ &= -9 \times 10^{-11}, \end{aligned} \quad (12)$$

where we have used $m_\tau = 1.78 \text{ GeV}$, $m_b = 4.3 \text{ GeV}$, and $m_t = 170 \text{ GeV}$ for the numerical estimate. The last term in this equation comes from the $k^\mu k^\nu$ part of the Z propagator. This last term with the sum over τ and b would have been the contribution from the top quark had we done the calculation with the full theory, i.e. without integrating out the top.

Altogether, treating the light u , d , and s quarks in the chiral limit, and neglecting the effect of hadronic mass scales, we find that in the Standard Model the leading contribution to the muon $g-2$ from the full set of fermionic triangle graphs in Fig. 2 represents a correction of $\sim -7\%$ to the dominant one-loop electroweak contribution.

Of course the limit $m_t \rightarrow \infty$ is quite far away from the real situation, and in the next section we will present exact expressions that will give corrections to this extreme limit.

6. The effective field theory framework is not very useful when non-leading effects in the large ratios of masses are also required. In that case it is simpler to do a direct calculation. The result corresponding to the diagrams in Fig. 2, in the unitary gauge, for a fixed fermion f in the triangle loop and without approximations has the form

$$\Delta a_\mu^{\text{Weak}}|_f = -\frac{G_F}{\sqrt{2}} \frac{m_\mu^2}{8\pi^2} \frac{\alpha}{\pi} Q_f^2 a_f \left\{ \mathcal{F}\left[\frac{m_f^2}{m_\mu^2}, \frac{M_Z^2}{m_\mu^2}\right] - \frac{m_f^2}{M_Z^2} \mathcal{G}\left[\frac{m_f^2}{m_\mu^2}, \frac{M_Z^2}{m_\mu^2}\right] \right\}, \quad (13)$$

where we have separated the contributions generated by the $g_{\mu\nu}$ term in the Z propagator (the \mathcal{F} -function) from those generated by the $k_\mu k_\nu$ term (the \mathcal{G} -function). As already mentioned earlier, it is the $k_\mu k_\nu$ term which also generates a divergent piece, independent of the fermion mass in the loop, and which here has been subtracted. (It cancels of course, once the sum over the fermions of a generation is made.) The functions \mathcal{F} and \mathcal{G} have rather compact Feynman parametric representations. With x and y the Feynman parameters of the triangle loop, and u, v , and w those of the other loop we have:

[¶]There are of course other operators relevant to other processes. [12]

$$\begin{aligned}
\mathcal{F}\left[\frac{m_f^2}{m_\mu^2}, \frac{M_Z^2}{m_\mu^2}\right] &= \int_0^1 dw \int_0^1 dv \int_0^1 du \int_0^1 dx \int_0^{1-x} dy \frac{8u^2v}{u^2v^2w^2 \frac{m_\mu^2}{M_Z^2} + \frac{u(1-v)}{y(1-y)} \frac{m_f^2}{M_Z^2} + (1-u)} \\
&\times \left\{ \frac{2x}{1-y} - 3(1+uvw) + \frac{u^3v^3w^3}{u^2v^2w^2 + \frac{u(1-v)}{y(1-y)} \frac{m_f^2}{m_\mu^2} + (1-u) \frac{M_Z^2}{m_\mu^2}} \right\}, \quad (14)
\end{aligned}$$

and

$$\begin{aligned}
\mathcal{G}\left[\frac{m_f^2}{m_\mu^2}, \frac{M_Z^2}{m_\mu^2}\right] &= \int_0^1 dw \int_0^1 dv \int_0^1 du \int_0^1 dx \int_0^{1-x} dy \frac{8u^2v}{u^2v^2w^2 \frac{m_\mu^2}{M_Z^2} + \frac{u(1-v)}{y(1-y)} \frac{m_f^2}{M_Z^2} + (1-u)} \\
&\times \left\{ -\frac{1+3uvw}{y(1-y)} + \frac{u^3v^3w^3}{u^2v^2w^2y(1-y) + u(1-v) \frac{m_f^2}{m_\mu^2} + (1-u)y(1-y) \frac{M_Z^2}{m_\mu^2}} \right\}. \quad (15)
\end{aligned}$$

The contribution from the \mathcal{G} -function to the muon $g-2$ is only sizeable for the t quark. It is in fact this piece which reproduces the leading $\log \frac{m_t^2}{M_Z^2}$ discussed earlier. We can now obtain the non-leading behaviour as well, with the result:

$$\Delta a_\mu^{\text{Weak}}[t]_{\text{eq.}(13)} = -\frac{G_F}{\sqrt{2}} \frac{m_\mu^2}{8\pi^2} \frac{\alpha}{\pi} \left\{ \frac{8}{3} \log \frac{m_t^2}{M_Z^2} + \frac{16}{3} - \frac{2}{9} \frac{M_Z^2}{m_t^2} \log \frac{m_t^2}{M_Z^2} + \mathcal{O}\left(\frac{M_Z^2}{m_t^2}\right) \right\}. \quad (16)$$

We can also use the expressions above to obtain a more accurate evaluation of the contributions to $\Delta a_\mu^{\text{Weak}}$ when the fermion f in the triangle loop is one of the three leptons: $f = e, \mu, \tau$. When the fermion f in the triangle loop is a quark, the expression in eq. (13) corresponds to the evaluation one obtains in the limit where the QCD gluonic interactions are neglected. If the quark in the triangle loop is a *heavy quark*: $f = c, b, t$, the QCD gluonic interactions can be treated perturbatively, and the corrections to the lowest-order estimate obtained, will be down by a typical factor $\alpha_s(\mu^2)/\pi$, where the mass scale μ runs between m_f and M_Z [13]. We can therefore use eq. (13) to reliably obtain a more elaborate estimate of the contribution to the muon $g-2$ from the third generation of fermions, with the result:

$$\Delta a_\mu^{\text{Weak}}[\tau, b, t]_{\text{eq.}(13)} = -8.2 \times 10^{-11}, \quad (17)$$

to be compared with our approximate estimate in eq. (12).

7. When the fermion f in the triangle loop is a *light quark*: $f = u, d, s$, the result obtained from a straightforward application of the expression in eq.(13) with either current algebra quark masses or constituent quark masses can be rather misleading. QCD perturbation theory is not justified in this case; and it would be erroneous to use it since, among other things, it neglects the fact that chiral symmetry is spontaneously broken. An appropriate way to discuss this problem is within the combined framework of chiral perturbation theory (χ PT) and the $1/N_c$ expansion. (For a recent review, where earlier references can also be found, see

e.g. ref. [14].) To lowest order in the chiral expansion in $U(3)_L \times U(3)_R$, the hadronic $Z\text{--}\gamma\text{--}\gamma$ interaction appears via the one Goldstone meson exchanges between the effective coupling

$$\mathcal{L}^{(2)} = \frac{-e}{2 \sin \vartheta_W \cos \vartheta_W} f_\pi \partial_\mu \left(\pi^0 + \frac{1}{\sqrt{3}} \eta_8 - \frac{1}{\sqrt{6}} \eta_0 \right) Z^\mu, \quad (18)$$

induced by the lowest $\mathcal{O}(p^2)$ chiral effective Lagrangian, and the effective $\mathcal{O}(p^4)$ coupling

$$\mathcal{L}_{ABJ} = \frac{\alpha}{\pi} \frac{N_c}{24 f_\pi} \left(\pi^0 + \frac{1}{\sqrt{3}} \eta_8 + 2 \sqrt{\frac{2}{3}} \eta_0 \right) \epsilon_{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma}; \quad (19)$$

i.e. the term in the Wess–Zumino Lagrangian which reproduces the Adler, Bell–Jackiw anomaly. The corresponding Feynman diagrams are shown in Fig.5. The evaluation of the contribution to the muon $g - 2$ from these diagrams, in the unitary gauge and in the chiral limit, leads to the result:

$$\Delta a_\mu^{\text{Weak}}[u, d, s]|_{\chi\text{PT}} = \frac{G_F}{\sqrt{2}} \frac{m_\mu^2}{8\pi^2} \frac{\alpha}{\pi} \left[\frac{4}{3} \log \frac{M_Z^2}{m_\mu^2} + \frac{4}{9} + \mathcal{O}\left(\frac{m_\mu^2}{M_Z^2} \log \frac{M_Z^2}{m_\mu^2}\right) \right] = 5.0 \times 10^{-11}, \quad (20)$$

where we have also subtracted the divergent piece generated by the $k_\mu k_\nu$ term in the Z propagator, which cancels with the corresponding pieces generated by the e , μ , and c fermionic loops. We wish to emphasize that to lowest non-trivial order in χPT , in the chiral limit and in the large N_c limit of QCD, this is an exact result. When added together with the contributions to $\Delta a_\mu^{\text{Weak}}|_f$ in eq. (13) from the leptons e , μ , and the c quark in the triangle loop, it gives as the result to the contribution to $\Delta a_\mu^{\text{Weak}}$ from the first and second generation of leptons and quarks:

$$\Delta a_\mu^{\text{Weak}}[e, d, u; \mu, s, c] = -8.7 \times 10^{-11}, \quad (21)$$

to be compared with our first estimate, the sum from eqs. (10) and (11).

8. We now want to discuss the sources of corrections to the above calculation and their possible estimate.

One source is the effect of the $\alpha_s(\mu^2)/\pi$ perturbative corrections to the lowest-order c , b , and t loop calculations which we have already mentioned. They are, in principle, under control and one does not expect them to drastically change our results.

The other sources of corrections concern the light-quark sector. Here, besides the obvious change in the Nambu–Goldstone propagator due to finite pseudoscalar masses, we expect chiral loop corrections due to the explicit breaking of the chiral symmetry, as well as corrections due to the contributions from higher order terms in the effective chiral Lagrangian. Chiral-loop corrections are suppressed in the $1/N_c$ expansion. We expect them to give corrections $\mathcal{O}(\frac{m_\eta^2}{16\pi^2 f_\pi^2})$, perhaps enhanced by chiral logarithmic factors. However, the most important effect to next to leading order in the $1/N_c$ expansion is likely to be the fact that, because of the $U(1)_A$ anomaly, the singlet η_0 particle acquires a large mass. The effect of this mass will be to damp the contribution of the η' to the muon $g - 2$ in eq. (20).

Concerning the effect due to higher order terms in the chiral expansion, it is possible to make estimates using models of the QCD low-energy effective action at large N_c , which have been developed during the last few years (see e.g. ref. [14].) The simplest version of these

models amounts in practice to giving a constituent mass M_Q to the u , d , and s quarks, and to modulate the axial–vector coupling of the constituent quarks with a constant g_A [15], [16]. Here, care must be taken, however, on the way constituent quark masses and the coupling g_A are introduced. As already mentioned, the VVA vertex in the triangle loop in Fig. 2 has an anomalous Ward identity: in the chiral limit the $VV\partial A$ vertex has a universal form which is, in particular, at the origin of the effective chiral realization discussed above. With k_μ the momentum flowing from the axial coupling, and $T^{\mu\alpha\beta}$ the full VVA vertex, we can decompose this vertex as follows:

$$T^{\mu\alpha\beta} = \frac{1}{k^2} k^\mu k_\rho T^{\rho\alpha\beta} + \left(T^{\mu\alpha\beta} - \frac{1}{k^2} k^\mu k_\rho T^{\rho\alpha\beta} \equiv R^{\mu\alpha\beta} \right). \quad (22)$$

The first term in the r.h.s., when modulated by the appropriate $\sum_f Q_f^2 a_f$ factor, with $f = u, d, s$ and including the N_c colour factor reproduces, in the chiral limit, the expression obtained in the effective chiral realization that we have already discussed (see eqs. (18) and (19).) This is an example of the ‘t Hooft anomaly matching condition [17] applied to QCD. The constituent chiral quark picture should then be applied to the second term only, i.e. the $R^{\mu\alpha\beta}$ tensor defined as the difference in eq. (22). (Notice that the contribution to the muon $g - 2$ from the R -tensor is gauge invariant.) This way, and using the values $M_Q \simeq (265 \pm 10) \text{ MeV}$ and $g_A \simeq 0.6 \pm 0.1$, we find a correction of at most $\simeq 30\%$ to the result in eq. (20). From this we conclude that 50% is a safe estimate of the size of the expected errors in the χ PT calculation in eq. (20).

Our final estimate of the leading (in the sense of eq. (5)) electroweak contributions from the full set of fermionic triangle loops in the Standard Model, from eqs. (17) and (21) is then:

$$\Delta a_\mu^{\text{Weak}}[e, d, u; \mu, s, c; \tau, b, t] = -(16.9 \pm 2.5) \times 10^{-11}. \quad (23)$$

This result, when added to the other leading two-loop electroweak corrections calculated in [11], corresponds to an overall estimate

$$\Delta a_\mu^{\text{Weak}} \simeq -(36.9 \pm 2.5) \times 10^{-11}; \quad (24)$$

i.e. a negative correction of $\sim 19\%$ to the lowest order electroweak contribution in eq. (4).

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Note Added

After completion of this work we have become aware of a paper by A. Czarnecki, B. Krause and W.J. Marciano, hep-ph/9506256, where a subset of the constant (i.e. non-log M/m enhanced, see eq. (5)) contributions has been calculated. These contributions originate mainly in m_t^2 -like corrections; they are negative and push the final result in eq. (24) to -45×10^{-11} .

References

- [1] J. Bailey *et al.*, Phys. Lett. B68 (1977) 191; F.J.M. Farley and E. Picasso, in *Quantum Electrodynamics*, ed. by T. Kinoshita, World Scientific, Singapore, 1990.
- [2] V.W. Hughes, in *Frontiers on High Energy Spin Physics*, ed. by T. Hasegawa *et al.*, Universal Academy Press, Tokyo, 1992.
- [3] B.E. Lautrup, A. Peterman, and E.de Rafael, Phys. Rep. 3C (1972) 193.
- [4] J. Calmet, S. Narison, M. Perrottet, and E. de Rafael, Rev. Mod. Phys. 49 (1977) 21.
- [5] T. Kinoshita and W.J. Marciano, in *Quantum Electrodynamics*, ed. by T. Kinoshita, World Scientific, Singapore, 1990.
- [6] S. Eidelman and F. Jegerlehner, preprint PSI-PR-95-1.
- [7] E. de Rafael, Phys. Lett. B322 (1994) 239.
- [8] M. Hayakawa, T. Kinoshita, and A.I. Sanda, preprint DPNU-94-21.
- [9] J. Bijnens, E. Pallante, and J. Prades, preprint NORDITA-95/36 N,P.
- [10] W.A. Bardeen, R. Gastmans, and B.E. Lautrup, Nucl. Phys. B46 (1972) 319.
- [11] T.V. Kukhto, E.A. Kuraev, A. Schiller, and Z.K. Silagadze, Nucl. Phys. B371 (1992) 567.
- [12] See, for instance, S. Peris and A. Santamaria, CERN-TH/95-21, and references therein. To appear in Nucl. Phys. B (1995).
- [13] S. Peris, Phys. Lett. B343 (1995) 339.
- [14] E. de Rafael, *Chiral Lagrangians and Kaon CP-Violation*, 1994 TASI-lectures, preprint CPT-95/P.3161
- [15] J. Bijnens, C. Bruno, and E. de Rafael, Nucl. Phys. B390 (1993) 501.
- [16] S. Peris and E. de Rafael, Phys. Lett. B309 (1993) 389.
- [17] G. 't Hooft, in *Recent Developments in Gauge Theories*, ed. G. 't Hooft et al. (Plenum N.Y., 1980).

Figure Captions

Fig.1 : Feynman diagrams which, in the Standard Model, give the lowest order electroweak contribution to the muon $g - 2$.

Fig.2 : Two-loop electroweak corrections induced by virtual fermionic triangle loops.

Fig.3 : Two-loop electroweak contributions, with virtual fermionic triangle loops, induced by a local four-fermion effective coupling.

Fig.4 : One-loop electroweak contribution induced by an effective $Z\text{--}\gamma\text{--}\gamma$ local coupling.

Fig.5 : Lowest order contribution in the effective chiral realization of QCD, which leads to a $Z\text{--}\gamma\text{--}\gamma$ hadronic contribution to the muon $g - 2$.

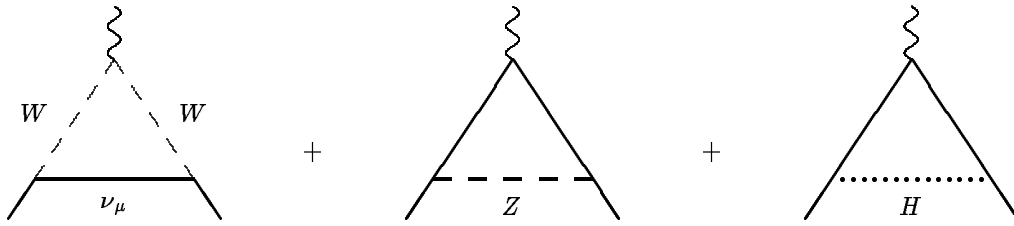


Fig. 1

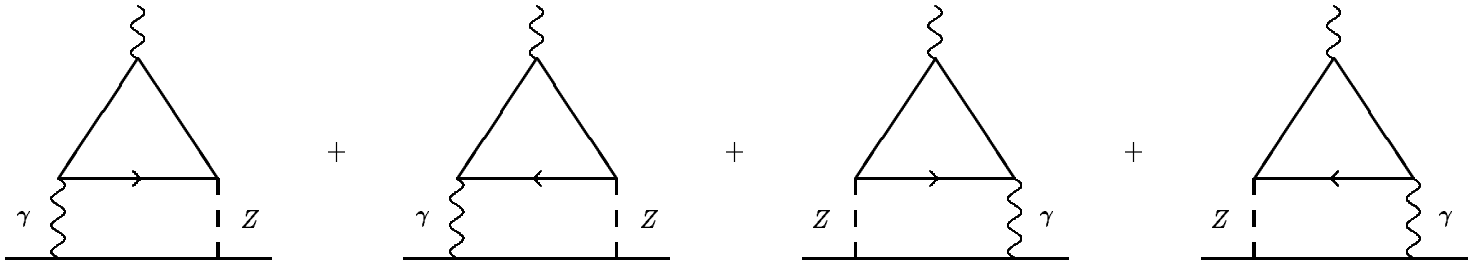


Fig. 2

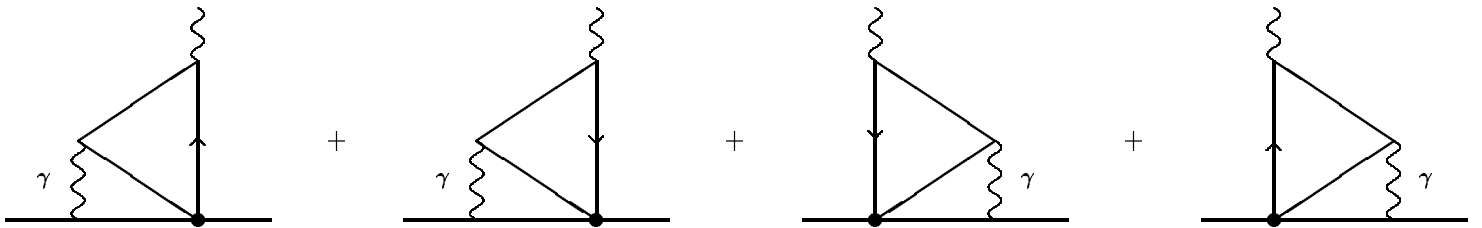


Fig. 3

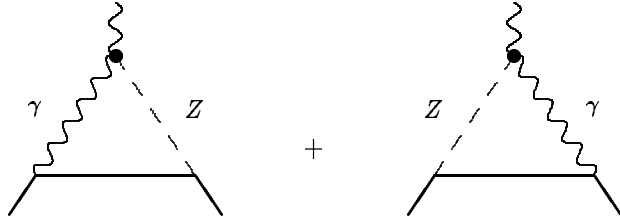


Fig. 4

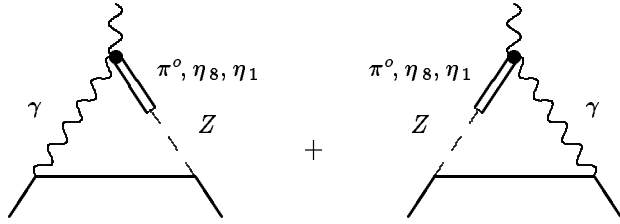


Fig. 5